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Comments on "Acoustic wave propagation through porous media revisited" [J. Acoust. Soc. Am. 100, 2949–2959 (1996)]

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In a 1996 paper by T. W. Geerits, published in this *Journal*, it was claimed that a new theory for acoustic wave propagation in porous media was derived. It will be shown that this claim is not justified. Correct application of the averaging procedures and correct interpretation of the dissipation leads to a one-to-one agreement with the Biot theory, which was published in this *Journal* in 1956 [M. A. Biot, J. Acoust. Soc. Am. **28**, 168–178, 179–191 (1956)]. © *1999 Acoustical Society of America.* [S0001-4966(98)00210-0]

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I. MACROSCOPIC EQUATIONS

We will compare the momentum equations and the stress–strain relations by Geerits (1996) (referred to by "G" in the following) and Biot (1956) (referred to by "B" in the following) for the case of isotropic porous media in absence of body forces and dissipation. The fluid momentum equation (G45) then becomes

$$\frac{\partial}{\partial x_i}\langle\sigma\rangle = m^{ff}\langle\dot{v}_i^f\rangle + m^{fs}\langle\dot{v}_i^s\rangle, \tag{1}$$

where σ is the microscopic fluid traction, and $\mathbf{v}^f$ and $\mathbf{v}^s$ are the microscopic velocities of the fluid and the solid, respectively. The averaging $\langle\cdots\rangle$ of the microscopic quantities takes place over the bulk volume V_b, consisting of pore space and solid grains. The parameters m^{ff} and m^{fs} are specific masses. We have substituted the identities for isotropic media $m_{ij}^{ff} = m^{ff}\delta_{ij}$ and $m_{ij}^{fs} = m^{fs}\delta_{ij}$ into the original equation (G45) to obtain (1). The dot over a variable denotes the time derivative of that variable.

In the Biot paper, macroscopic fluid and solid displacements $\mathbf{U}$ and $\mathbf{u}$ are defined. They are intrinsic variables, which means that the averaging has taken place over the fluid space ϕV_b and the solid space $(1-\phi)V_b$, respectively, where ϕ is the porosity. Thus (1) becomes in its linearized form:

$$\frac{\partial}{\partial x_i}\langle\sigma\rangle = \phi m^{ff}\ddot{U}_i + (1-\phi)m^{fs}\ddot{u}_i. \tag{2}$$

The corresponding relation in the Biot paper is given by (B3.21):

$$\frac{\partial s}{\partial x_i} = \rho_{22}\ddot{U}_i + \rho_{12}\ddot{u}_i, \tag{3}$$

where the scalar s represents the forces per unit bulk area acting on the fluid part of each face of a unit cube of porous material. Therefore, $s = \langle\sigma\rangle$. Furthermore, denoting the fluid density by ρ_f, Biot defines $\rho_{22} = \phi\rho_f - \rho_{12}$, where ρ_{12} is the mass coupling parameter. From (2) and (3) it can easily be seen that

$$m^{ff} = \frac{\rho_{22}}{\phi}, \quad m^{fs} = \frac{\rho_{12}}{(1-\phi)}. \tag{4}$$

In a similar way the solid momentum equation (G47) in the paper by Geerits can be compared with its Biot equivalent (B3.21), and it follows that for the specific masses m^{ss} and m^{sf} it can be written that

$$m^{ss} = \frac{\rho_{11}}{(1-\phi)}, \quad m^{sf} = \frac{\rho_{12}}{\phi}, \tag{5}$$

where, denoting the solid density by ρ_s, Biot defines $\rho_{11} = (1-\phi)\rho_s - \rho_{12}$. We immediately notice that m^{fs} and m^{sf} are not identical as is stated by Geerits in his symmetry relations (G64). Essentially, this is due to the fact that the velocities are averaged over the bulk volume instead of over the intrinsic volume. This averaging also gives rise to misleading results in the expression for the kinetic energy (G75). We will evaluate this expression for the case of a porous medium consisting of an ensemble of parallel tubes. We will assume that the porous medium is rigid, i.e., $v_i^s = 0$:

$$E_{\text{kin}} = \tfrac{1}{2}m^{ff}\langle v_i^f\rangle\langle v_i^f\rangle, \tag{6}$$

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where $m^{ff}=\rho_f-m^{sf}$. This means that the kinetic energy is given by $\frac{1}{2}\rho_f(\phi\dot{U})^2$, where $\dot{U}$ is the uniform fluid velocity in the direction of the tubes. Obviously, the correct expression is given by $\frac{1}{2}\phi\rho_f\dot{U}^2$, as becomes clear from Biot's equation (B3.2).

For the stress–strain relations, we follow the same procedure. Omitting the source density, (G46) becomes

$$\frac{\partial}{\partial x_i}\langle v_i^f\rangle=k^{ff}\langle\dot{\sigma}\rangle+\frac{1}{3}k^{fs}\langle\dot{\sigma}_{kk}^s\rangle, \tag{7}$$

where σ_{ij}^s is the microscopic solid stress tensor, and k^{ff} and k^{fs} are scalar compressibilities.

In the Biot paper, the stress–stain relations (B2.12) are written as

$$\tau_{ij}=2Ge_{ij}+Ae_{kk}\delta_{ij}+Q\epsilon_{kk}\delta_{ij},$$
$$s=Qe_{kk}+R\epsilon_{kk}, \tag{8}$$

where

$$e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\quad \epsilon_{ij}=\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right). \tag{9}$$

The tensor τ_{ij} represents the forces per unit bulk area acting on the solid parts of each face of a unit cube of porous material. Furthermore A, Q, and R are generalized elastic constants which can be related to such measurable quantities as the fluid bulk modulus K_f, the matrix shear modulus G, and to K_b and K_s, which are the bulk moduli of the porous frame and individual grains respectively. From (8) it can be derived that

$$\dot{\epsilon}_{kk}=\frac{D}{DR-Q^2}\dot{s}-\frac{1}{3}\frac{Q}{DR-Q^2}\dot{\tau}_{kk}, \tag{10}$$

where $3D=2G+3A$. Relation (10) can now be compared with (7), where we notice that the LHS of (7) equals $\phi\dot{\epsilon}_{kk}$, that $\langle\dot{\sigma}\rangle=\dot{s}$, and that $\dot{\tau}_{kk}=\langle\dot{\sigma}_{kk}^s\rangle$. It then follows that

$$k^{ff}=\frac{D\phi}{DR-Q^2},\quad k^{fs}=\frac{-Q\phi}{DR-Q^2}. \tag{11}$$

In a similar way, the solid equation (G48) can be compared with its Biot equivalent (B2.12). Then it is found that $k^{sf}=-Q(1-\phi)/(DR-Q^2)$. This means that also the compressibility parameters introduced by Geerits can directly be expressed in terms of Biot parameters. Moreover, the symmetry relation (G65) $k^{fs}=k^{sf}$ does not hold.

II. DISSIPATION

In order to describe dissipation, Geerits introduces on the macroscale the parameters R^{fs} and R^{sf}. These parameters represent the so-called "losses of the interface type," also referred to as "inertial losses," or "losses due to tortuosity." This is not a realistic concept. The correct concept is based on viscosity, and was introduced by Biot (1956), who defined a frequency-dependent friction factor $b(\omega)$. At low frequencies, $b(\omega)=b_0=\eta\phi^2/k_0$, where η is the fluid viscosity, and k_0 is the permeability. At higher frequencies, when the viscous skin depth $\delta=\sqrt{2\eta/\omega\rho_f}$ decreases, inertia will become dominant over viscosity. However, inertia has nothing to do with the damping of the waves. At all frequencies, all damping is exclusively caused by viscosity. This can elegantly be illustrated for plane wave propagation in a rigid tortuous porous medium filled with a viscous compressible fluid. For propagation in the z direction, an $\exp i(\omega t-kz)$ dependence for the relevant variables is introduced, where $k=\omega/c$ is the wave number and c is the wave velocity. The momentum equation for the fluid is now given by (Johnson and Plona, 1982):

$$i\omega\rho_f\left[\alpha_\infty-\frac{ib(\omega)}{\omega\phi\rho_f}\right]\hat{w}=ik\hat{p}. \tag{12}$$

Here, α_∞ denotes the tortuosity. It is a purely geometrical quantity, independent of fluid or solid densities, and its value is always greater than or equal to unity. The fluid velocity and pressure are denoted by $\hat{w}$ and $\hat{p}$, respectively. Combination of (12) with the mass conservation law yields the dispersion relation:

$$k=\frac{\omega}{c_f}\sqrt{\alpha_\infty-\frac{ib(\omega)}{\phi\rho_f\omega}}, \tag{13}$$

where c_f is the speed of sound in the fluid. The damping factor d is defined as $-\mathrm{Im}(k)$. This means that if viscosity is neglected, i.e., $b(\omega)=0$, the waves will not be damped and propagate with a characteristic wave speed $c_{\mathrm{ref}}=c_f/\sqrt{\alpha_\infty}$. If on the other hand tortuosity is neglected, i.e., $\alpha_\infty=1$, the damping of the waves definitely does not disappear. In this case (13) describes the wave propagation and damping in a fluid cylinder with fixed walls. We therefore conclude that damping is a viscosity-based mechanism, and not a tortuosity-based mechanism. This also means that for the parameters R^{sf} and R^{fs} in the paper by Geerits, it can simply be written that

$$R^{sf}=b(\omega)/\phi,\quad R^{fs}=b(\omega)/(1-\phi). \tag{14}$$

Also in this case, the symmetry $R^{sf}=R^{fs}$ claimed by Geerits does not hold. Moreover, we notice that the dissipation parameters can simply be written in terms of Biot parameters. Detailed expressions for the friction factor $b(\omega)$ were given by Johnson *et al.* (1987) and Smeulders *et al.* (1992).

III. CONCLUSIONS

We have shown that the mass and compressibility parameters introduced by Geerits can directly be expressed in terms of existing Biot parameters. The averaging procedures used by Geerits give rise to misleading symmetry relations. The newly introduced dissipation parameters describe in fact a viscous mechanism, discussed previously by, among others, Biot (1956), Auriault *et al.* (1985), and Johnson *et al.* (1987).

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